THE EARTH

AND ITS CHIEF MOTIONS,

AND

THE TANGENT INDEX.

BY

JOHN HAYWOOD,

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CHAPTER I.

OUR WORLD.

The object of these lines is to furnish some help to those who are studying the elements of astronomy, by assisting them to comprehend the most important of our astronomical relations.

We find ourselves inhabitants of a world which to our senses is infinite, and fixed in its place. When we attempt to explore it, we find ourselves held within certain limits. Some of these limits—seas, deserts, wildernesses—human ingenuity and perseverance enable the explorer to break through Other limits we know can never be passed over. Still others are doubtful. Thus we never expect to penetrate deep into the earth. We cannot rise much above its surface. Travelers ascending mountains, or navigating the air in balloons, imperil their lives, and not infrequently lose them. At most, the height to be attained in either of these ways is small.

Again, the frozen zones seem to present impassable barriers to the daring navigators who have attempted to pass them; and though many lives have been lost, and much treasure has been expended in the attempts to explore these regions, they remain, to a great extent, unknown. Hence our knowledge of our world is imperfect, and doubtless there will always be more of it unknown than known.

With respect to other worlds, they are more inaccessible than the frozen zones or the interior of the earth. A great gulf, which no man can pass over, separates us from them. It might well seem that such distant and inaccessible worlds were not proper objects of investigation. On the contrary, this very study, astronomy, has

helped us in carrying forward the study of our own world. Without astronomy, science and civilization would necessarily be much more imperfect than they are.

The science of astronomy is possible to us because we have the power of vision, and what is of more importance, the power of reasoning, by which, on the comparison of known truths, we deduce with certainty various connected truths. The certainty of these deductions is so great that we unhesitatingly correct our erroneous ideas derived from the more imperfect testimony of the senses. Thus the traveler in the desert sees what seems to be sheets of water, green herbage, trees, and buildings, but judges the appearances an illusion. Thus when we are on the water in a vessel, we cannot at once determine whether the motion we see is a motion of our vessel or of other vessels or objects near by. Sometimes when we are on a railroad train entering or leaving the station, we notice an illusion of the same kind. But the most complete and ineradicable illusion in our experience is exposed and corrected in the study of astronomy; an illusion that is absolutely universal, and, do what we will, adheres to us so tenaciously that we can only correct it in thought, in judgment—the illusion that this world is at rest and fixed in space; and that up and down are invariable directions in space. So deep-seated and unchangeable is the conviction derived from these appearances that astronomers do not try to change the language ordinarily used in speaking of the various astronomical phenomena, the rising and setting of the sun, moon, and stars, etc.

CHAPTER II.

THE EARTH A SPHERE.

To the eye, the earth presents the appearance of a boundless plain, diversified with mountains, hills, and valleys, plains and seas. In our experience, all objects rest upon something. If the support upon which an object rests be taken away, the object falls, descends toward the ground until it finds a support, either on the ground itself or upon something which rests upon the ground. This universal experience leads us naturally, when discussing the form of the earth, to inquire what supports the earth, which itself serves as the support of all things belonging to it? It requires careful investigations to make certain, or to correct, our impressions and natural convictions on these points.

The proofs of the spherical form of the earth are sufficiently numerous and convincing when we bring our reasoning powers to bear upon them. The manner of the disappearance of a ship sailing away from us is to the purpose. The body of the ship disappears below the surface of the water, while the upper parts, the masts and sails, are still visible; then they, too, disappear below the water-line. In like manner, when a ship is coming toward us, we see the top-sails first, then the lower sails and the hull successively heave in sight.

The shadow of the earth upon the moon when the latter is in eclipse also exhibits the same truth. We reason in this case, and in the preceding cases, that the appearances presented, (the phenomena), compel us to conclude that the earth is spherical. Other considerations, which we will not stop to present, lead us to the same conclusion.

Now, admitting the sphericity of the earth, what do the words up and down mean? Travelers in all parts of the world find the appearances of up and down the same as we. We may suppose a traveler on the opposite side of this globe — the earth — from us,

and may conceive a straight line, a diameter, passed through the center of the earth from our place to the opposite place. Now, evidently, down, to this traveler will be in exactly the opposite direction in space to that indicated by down with us. We conclude, then, that the terms down and up relate to the earth, that down means toward the earth, toward its center, and up means away from the earth.

Also, when we see unsupported objects fall to the ground, whereever on the earth the observation is made, we conclude that by falling we mean really a moving toward the earth; that bodies are affected by some force which we call gravity, which urges them toward the earth. We may carry our conclusions further. If we could in some way place ourselves somewhere in space, away from this world and from any other worlds, we may conclude that there would be no down nor up, that there would be no falling. Dwellers upon the earth, travelers wherever they are, find the earth, with all that belongs to it, separated from all other worlds and objects by an inconceivable distance. The heavens seem as distant from us in one part of the earth as in another. They seem above us in one part the same as in another. Indeed, we ourselves can here at home make these observations for ourselves, since we are carried around the earth in its diurnal motion, as will be more fully shown hereafter; and we see a different heaven above us by night and by day. Therefore the earth is itself in the situation supposed of a body out in space, and so far separated from other worlds that it is so little affected by them that we may, for our present discussion, consider it quite unaffected; therefore the earth needs no foundation—that is, it is so far removed from other worlds that it keeps its place without any support.

CHAPTER III.

THE EARTH A SPHERE.—Continued.

It is therefore correct to say that the earth is but little affected by other worlds. It is, in fact, urged both toward the moon and toward the sun; and, indeed, toward every member of the solar system, if not toward every star in the sky. But the intensity of gravity toward these distant bodies, even the nearest of them—the moon—is feeble, comparatively. This can be readily shown. A mass of any material, lead for instance, weighing 3,600 pounds on the earth, will weigh, at the distance of the moon—sixty times as far from the center of the earth—only one pound; that is, the same muscular force exerted in holding up one pound here on the earth's surface, would, at the distance of the moon, hold up the mass of lead above mentioned, so far as it is affected by terrestial gravity.

It will be noticed that the term, weight, used in this connection, is ambiguous. Thus we have weighing machines in which the material to be weighed is balanced by counterpoises; and we have machines in which the weight is held up by steel springs, and the amount of weight is measured by the compression of the springs. If the mass of lead were weighed by a machine of the former kind, it would weigh as much at the distance of the moon as on the earth's surface, since the gravity of the counterpoises would be affected by the distance in the same proportion as that of the mass to be weighed; but if the mass of lead were weighed by a spring balance at the distance of the moon, it would show a weight of one pound only. The increased distance, which causes a diminution of the intensity of gravity, of course does not affect the elasticity of the spring.

Again, a body, as a ball of lead, near the earth, will fall from a state of rest by gravity, in one second, sixteen and one-twelfth feet —that is, by terrestial gravity of the intensity we experience on the earth; but at the distance of the moon it would, under the same conditions, fall only 0.05 of an inch.

Again, it is known that solar gravity is more than 300,000 times greater than terrestial gravity under the same conditions of distance. A piece of lead weighing a pound on the earth would, at the sun's surface, weigh twenty-seven and a half pounds in a spring balance. It would fall from rest by solar gravity near the sun's surface, 440 feet in one second. But at the earth's distance from the sun, the solar gravity is so enfeebled that a mass weighing on the earth one ton, would, by solar gravity, at that distance, weigh by a spring balance only one pound; and it would fall toward the sun from a state of rest only one-tenth of an inch in one second.

It will be seen from these statements that the proposition that the earth is in the situation of a body removed from the influence of the gravity of other worlds is substantially proved. The important fact that this enfeebled solar gravity is counterbalanced by centrifugal force generated by the revolution of the earth about the sun, is considered further along.

According to the statements made above, it will be readily seen that the moon is also a world, in some sort connected with the earth, but so remote that terrestial gravity is only sufficient to keep it from abandoning the earth as it moves on its course; and we see here, also, the case of a world holding its place in space without support, and needing none.

CHAPTER IV.

ROTATION OF THE EARTH UPON ITS AXIS.

Having found the earth a sphere, and holding its place in space and unsupported, and needing no support, it is much easier to see that it may turn on its axis. There are many observed facts which prove the diurnal motion of the earth. It will be sufficient to present two of these most easily comprehended, and leading most directly to the conclusion.

The whole heavens seem to revolve around the earth every day. Every star, the sun and moon, are carried about the earth much as though they were attached to the inner surface of an immense hollow sphere which surrounds the earth, and which revolves upon its axis. This is the appearance the heavens present, and this was the first thought of the ancient astronomers. But we now know that these different celestial bodies are not attached to the surface of a sphere; that they are very unequally distant from us; that all of them are at such great distances that it is inconceivable that they should revolve about the earth in one day.

Another fact, not so obvious, is that the equatorial radius of the earth is about thirteen miles longer than the polar radius. This has been fully established by surveys made in different countries by the most accomplished engineers, and with the best surveying apparatus modern science and art have been able to furnish. Admitting this fact, we are compelled to admit the rotation of the earth, since if the waters of the equatorial seas were not held to the equatorial parts of the earth by some force to counteract gravity, they would flow to the north and south by the force of gravity, and form seas about the poles deep enough to bring the earth to the form of a sphere, except the enormous land elevations along the equator formed by the retreat of the waters. The centrifugal force generated by the rotation of the earth upon its axis just supplies this needed force; and we conclude the form of the earth to be slightly spheroidal,

being the form required to bring the force of gravity and the centrifugal force on the different parts of the earth's surface to an equilibrium.

Reasoning on these facts, we conclude that the earth turns on its axis every day, thus bringing about the succession of day and night.

CHAPTER V.

THE ANNUAL MOTION OF THE EARTH.

Besides the vicissitudes of day and night, which we ascribe to the revolution of the earth on its axis, we find a continual change of seasons. This the earth's diurnal motion does not account for. But by careful watching through the year, and year after year, we see that the sun moves to the north and back to the south regularly each year ; that when the sun is far north it is summer, and when it is far south it is winter in our northern hemisphere. To those living in the southern hemisphere the seasons are just opposite ; so that summer in the one hemisphere is at the time it is winter in the other. The change of seasons, therefore, we find is caused by the change of the sun's place to the north or south.

It is found convenient to refer the sun's place in this respect to the equator. Thus, to one living on the equator, the sun is half the year north of him, and the other half of the year south ; and twice in the year, March 21st and September 22d, it is on the equator—that is, on those days, the sun, in its apparent diurnal motion, rises exactly in the east, at noon is exactly over head, and sets exactly in the west—in other words, the sun's apparent path on these days coincides with the plane of the earth's equator. This path is called the celestial equator. It may also be conceived as a great circle of the heavens marked on the sky by an equatorial radius of the earth prolonged to the sky. Thus if this radius were a pencil, and the sky were a solid surface, there would be traced on the spherical surface, as the earth turned on its axis, a great circle of the celestial sphere, the celestial equator or equinoctial. Although there is not in fact any such visible circle on the sky as described, yet it is conceived of, and the sun's place from day to day is determined with reference to it. The distance of the sun from this circle at any time is called its declination. It is estimated in circular measure – that is, in degrees, etc. It will be noticed that declination has much the

same meaning as latitude, used to indicate the situation of a place on the earth ; also, it may be here stated that the places of the other celestial bodies, the moon, the planets, and the stars, are referred to the equinoctial ; the word declination being used still with the same meaning ; namely, the distance of a body north or south of the equinoctial.

While this change of the sun's declination is taking place, we shall, if watchful, observe also a change in our visible heavens. Many stars and groups of stars (constellations) which we noticed in March, are not visible in September ; or, if visible, are in a different part of the sky. Noticing more closely, we see a constant shifting of the constellations to the west, in a constant, endless procession, so that in a year's time they are seen returned to the position occupied at the beginning of the year.

We may reason on these phenomena as we did in the matter of the diurnal motion. Instead of supposing the stars attached to the inner surface of an immense hollow sphere, we say they are at different distances ; and in any case they are too distant to move through such a circuit in one year ; therefore, we ascribe the phenomena to the motion of the sun, a much nearer body, eastward in a path about the earth. This path careful observation shows to be a great circle of the heavens called the ecliptic. It intersects the equinoctial at two opposite points of the heavens called the equinoxes. The sun is at the one, the vernal equinox, on March 21st, and is then going northward in declination. It is at the other equinox, the autumnal, about September 22d, at that time going south. On the day about midway between these two dates, June 21st, the sun is at the point of the ecliptic having the greatest northern declination, about twenty-three and a half degrees. This point is called the summer solstice. Also, midway between the autumnal equinox and the vernal, the sun reaches its greatest southern declination, twenty-three and a half degrees. This point is called the winter solstice. The sun is at this point of the ecliptic about December 21st. This angular number, twenty-three and one half degrees, measures the angle included between the two circles, the equator and the ecliptic. It is called the obliquity of the ecliptic.

The circumference of the ecliptic is divided into twelve parts of

thirty degrees each, called signs. These signs have names given to them in the early history of astronomy. Thus, beginning at the vernal equinox, the first division eastward is called Aries, the second Taurus, etc. These names are found in all almanacs. The two equinoxes, therefore, are at the beginning of Aries and Libra; the two solstices at the beginning of Cancer and Capricorn.

CHAPTER VI.

THE ANNUAL MOTION OF THE EARTH.—Continued.

The last two names above, Cancer and Capricorn, are also given to the two circles conceived as drawn on the earth as boundaries of the torrid zone. The significance of these two names is readily seen. Thus on June 21st, when the sun is at the summer solstice, and having a declination of twenty-three and a half degrees north, it is directly over a point on the earth having a latitude of twenty-three and a half degrees north; and to a person on any point on the earth having this latitude, the sun at noon would be directly overhead. A circle, therefore, drawn around the earth parallel to the equator, and at a latitude of twenty-three and a half degrees north, passes through all the points on the earth at which the sun is vertical at noon on June 21st, when it is farthest north. This limit is taken as the northern boundary of the torrid zone. The circle is called tropic because the sun now stops (solstice) going north, and turns (tropic) southward.

In the same way we see why the southern boundary of the torrid zone is called the Tropic of Capricorn. The sun, on December 21st, as at the winter solstice, and in the course of its diurnal motion that day is directly overhead at noon to all points of the earth that are situated on a parallel to the equator, at a distance of twenty-three and a half degrees south of it. At this time the sun begins to return toward the north.

The two frigid zones are bounded by the two polar circles. These are small circles drawn around the poles, twenty-three and a half degrees from the poles. The reason for thus bounding these zones will be seen from the following considerations: When the sun is at the summer solstice, twenty-three and a half degrees north of the equinoctial, it will shine over all the north polar region to the distance of twenty-three and a half degrees, from the pole, and it will not set on that day to any part of this limited portion of the

earth's surface. To one standing at the pole, the sun would be seen to be twenty-three and a half degrees above the horizon, and to move around the horizon, keeping the same height above it, and making a complete circuit in twenty-four hours. In fact, it has been constantly in sight since March 21st, at which date it came up to the horizon, or rose. It came up slowly, higher and higher, still making its complete circuit every twenty four hours, till June 21st, or the summer solstice. Then it begins slowly to get lower, but all the time continuing its daily circuit, till September 22d, when it sets, to remain out of sight while the sun is passing from the autumnal equinox to the winter solstice, and on to the vernal equinox. Thus, at the pole, there is but one day and one night in a year, each six months long. At the south pole the appearances are the same, but reversed—that is, while it is day at one pole, it is night at the other.

If we change our station from the pole, and move down toward the polar circle, the phenomena of day and night change gradually to what we ordinarily see here in the temperate zone—that is, there will be, beginning at the vernal equinox, a rising and setting of the sun, the days growing longer as the season advances; then the sun will just skim the northern horizon at what might be called midnight; then the sun will remain continually above the horizon for days, or weeks, or months, as we are nearer the pole, until the summer solstice, and the same length of time after the solstice, when the sun will, at the hour of the day corresponding to midnight, just touch the northern horizon. Then, as the sun advances to the autumnal equinox, the succession of day and night is resumed, the days becoming shorter and the nights longer, until sometime after the autumnal equinox, the sun will, at noon, just come up to the southern horizon. Then it ceases to rise, and the night continues unbroken for a length of time about equal to the time the day continued unbroken in the opposite part of the year—that is, till some time after the winter solstice, when the sun is again seen in the southern horizon. Then the succession of day and night begins again, the days at first very short, then longer, till the vernal equinox, when the days and nights are equal. The days from this time go on lengthening, as before described.

But on the polar circle, at the time of the summer solstice, the

sun just comes down to the northern horizon at midnight; and then without going down out of sight, begins, in its diurnal motion, to rise as it moves around to the south where, at noon, it is forty-seven degrees high. Thus at all places on the polar circle, during a whole day at the summer solstice, the sun does not set, and for a whole day at the winter solstice the sun does not rise. This circle, therefore, forms the boundary of the frigid zone—the north frigid zone about the north pole, and the south frigid zone about the south pole.

The two temperate zones are bounded on the side toward the equator by the tropics; and by the polar circles on the side toward the poles.

This discussion shows how a knowledge of astronomy helps to make the facts of geography plain.

CHAPTER VII.

THE ANNUAL MOTION OF THE EARTH.—CONTINUED.

The phenomena of the change of the seasons are accounted for, as we have before seen, by an annual motion of the sun about the earth in a great circle, the ecliptic, inclined to the equinoctial twenty-three and a half degrees. This motion, therefore, we call the sun's apparent annual motion. But astronomy teaches that the earth moves annually about the sun. Let us see on what ground. The question is more complex than that relating to the diurnal motion of the earth. An experiment in physics will help to an understanding of the subject. Let us take two stones, two pieces of brick, or better, two balls such as are used in playing ball, or better still, two leaden balls of suitable weight. Fasten them to the ends of a light rod of convenient length. Now we can support the rod on the edge of a knife in such a way that the two balls—we will call them bodies—will balance. If the two bodies are equal, the support will be under the middle point of the rod. If they are unequal, the support will be nearer the heavier body. In either case, the point at which they balance is called their common centre of gravity. Now, tie a string to the rod at this point, and suspend the apparatus at a convenient height from a nail or hook in the ceiling of the room. Then set the bodies to whirling about each other. It will be seen at once that each of them moves in a circle, and that the common centre of gravity is the common centre of both circles.

Another way to exhibit this law of revolution, and in some respects a better way, is to tie the two bodies to the ends of a string of sufficient length, and suspend them side by side from a hook in the ceiling. Now twist the strings together evenly and to such a degree that the effort to untwist when the bodies are set free will cause a fair velocity of revolution. Then the two bodies will swing apart more and more widely as the swiftness of revolution increases,

while at the same time they are drawn toward each other by the action of the strings and their gravity toward the earth. Now, whatever be the velocity of revolution, and their distance apart, it will be seen that they revolve about their common centre of gravity. This will be the case whether the bodies are equal or unequal; and, further, it will evidently be true whatever be their size, and if they are urged toward each other by their mutual gravity. Let, then, the two bodies be two worlds, situated in space at a suitable distance from each other and from all other worlds, so that they can move freely and not be swayed by any force but the mutual gravity which urges the two toward each other; and let them be made to revolve with sufficient velocity to generate a centrifugal force great enough to counterbalance their mutual gravity; then will the two worlds revolve about their common centre of gravity.

If we should observe this motion from a position on one of these worlds, it would appear to us that our world was stationary, and that the other world is revolving around our world. But we should know from the principle proved above, that in fact, they were revolving around their common centre of gravity. If these two worlds were of equal weight (mass), the centre would be midway between them. But if one had a very much greater mass than the other, it would be proper to say that the smaller world revolves about the larger, rather than the larger revolves about the smaller. But the fact is fully established that the mass of the sun is more than 300,000 times larger than the mass of the earth, as stated heretofore; therefore we say the earth revolves around the sun. On account of this enormous inequality in the mass of the sun and earth, we could not see any motion in the sun if we were ever so favorably situated to see it. If the mass of the earth were very much greater than it is, if all the planets were joined to the earth,—some of them are several hundred times greater than the earth,—still we should see this one great planet revolving around the sun, and still the sun's motion would be too inconsiderable to be noticed; so wonderfully large a world is our sun.

NOTE A.

It is known that the sun has a motion of rotation on its axis in about twenty-five and one third days, and this motion is readily ob-

served. This is not considered in the discussion; only the motion of displacement need be considered, and that is too little to be noticed, as stated above.

NOTE B.

In discussing the motion of two bodies about their common centre of gravity, the supposition was made that the bodies be removed from the influence of other bodies. Also that they be placed at a suitable distance from each other. The first supposition cannot be true of any two bodies in the solar system; as, for instance, the Sun and Earth. Thus the earth gravitates towards the other planets; the intensity of the force being different with the varying distance of the disturbing body. But its greatest value is small compared with the solar gravity. Yet the effect is great enough to be noticed and taken account of by the astronomers; and in preparing an ephemeris of a planet, as the earth, the disturbance caused by each of the other planets is computed and allowance is made. These disturbances are called perturbations.

With regard to the distance of two bodies which are supposed to constitute a system, it may be stated, that theoretically, two material points, that is things having gravity but not sensible magnitude, may form a system and perform their revolutions about each other at any distance however small. But bodies, having magnitude, need to be at a suitable distance from each other. It is obvious that the two bodies at their nearest approach must be at a distance greater than the sum of the radii of the two spheres. If the nearer parts were to collide, the system would of course collapse. But if the bodies approached closely without colliding, the unequal gravity of the nearer and remoter parts of the bodies would cause perturbations of sufficient magnitude perhaps to bring the system to an end. This is clearly the case when the bodies are two worlds. Thus the moon, as distant as it is, acts so unequally upon the nearer and more remote parts of the earth, that many interesting effects are produced. The most noticeable effect is produced upon the ocean, as would be naturally expected. The tides of the ocean, in their flow and ebb, indicate this disturbance. The extent of the rise and fall is greater as the moon is nearer. It is probable that the tides would be so great if the moon was at half its present distance from the earth, that our

world would be a desolate, uninhabited waste, the immense tide waves sweeping over the continents twice a day.

Other effects even more disastrous would result from bringing these bodies too near together. Thus in such immense masses of matter, cohesion is a force inferior to gravity ; hence the superior gravity of the earth would not only more than counterbalance the lunar gravity of the parts next to the earth ; but might even be in excess sufficient to break loose from the moon, mountains and other vast masses and cause them to fall upon the earth ; and thus the moon might be literally torn to pieces, and the earth might be overwhelmed by the wreck falling upon it.

NOTE C.

When two bodies revolve about their common centre of gravity, it is evident that the form of the paths described, (the orbits), will depend in part upon the nature of the force which urges them toward each other. Thus when two bodies are attached to the ends of a rod, and made to revolve, each body describes a circle about the common centre of gravity. If the masses of the two bodies are greatly disproportioned, as they are for instance in the case of the Earth and Sun, we say the smaller body describes a circle about the larger. When two bodies in space form a system, the form of the orbit described will depend upon the intensity of the mutual gravity, and upon the direction and velocity of the planet; when the direction of the motion is nearly perpendicular to the line joining the planet and the Sun, (the radius vector); and the mutual gravity is almost exactly balanced by centrifugal force; the orbit is nearly a circle. It is an ellipse of small eccentricity. This is the case with the earth and the other planets. It is conceivable that a planet revolve about the sun in an exact circle.

So the orbit under other conditions may be an ellipse of greater eccentricity, and even a parabola ; as is the case with some comets. Still another possible form of the orbit is the hyperbola. But in any case of a body urged towards the Sun by gravity, whatever be the velocity, and whatever be the direction, excepting motion in a direct line towards or from the Sun, it will move in an orbit of one or another of the kinds named. These curves belong to one class called Conic Sections.

CHAPTER VIII.

THE TANGENT INDEX.

We are now prepared to consider the questions, at what rate is the earth moving in its orbit? and, what is the direction in space of the earth's motion at this moment?

The earth's distance from the sun is about 92,000,000 miles. The orbit of the earth, its path, is nearly a circle—it is slightly elliptical. Counting it a circle, we multiply twice 92,000,000 by 3.1416 to find the circumference of the circle. This product is the distance the earth travels each year. It is nearly 580,000,000 miles. As the orbit is elliptical, the earth moves some faster when nearest the sun, about January 1st, and a little slower when farthest distant, about July 1st. Disregarding these inequalities, which are small, we find the earth moves about 1,500,000 miles each day. This gives a velocity of about 67,000 miles per hour; of 1,100 miles per minute, and nearly 18½ miles per second. This velocity far exceeds that of the swiftest moving objects which usually occur to us in making comparisons. A railway train of cars can be made to move one mile in a minute. The earth moves 1,100 times as fast. A cannon-ball may have, at the instant of discharge, a velocity of about 2,000 feet per second. The earth moves 50 times as fast. An illustration is sometimes given in this way: Suppose the cannon to be discharged in the direction of the earth's motion. While its velocity seems to us so great, yet, in fact, there has been added to its velocity, which it had in common with the earth, a small percentage—that is, its velocity is increased from 100,000 feet to 102,000 feet per second. On the other hand, if the cannon be discharged in the opposite direction; while to us it seems to be moving swiftly in that direction, it is really still going along in the same direction as the earth, but with a velocity relatively very little less—that is, the ball is going in the same direction as the earth 98,000 feet per second, while the cannon moves along with the earth 100,000 feet per second. Moreover if in this

last case the direction of discharge takes in some obstruction, as the wall of a fort or the side of a ship, at a suitable distance, the ball seems to us to dash against the obstacle; but, in fact, the obstacle dashes against the ball with a speed equal to their difference in velocity—that is 2,000 feet per second.

In respect to the direction of the earth's motion in its orbit, the general statement is that the earth, with all the planets, moves round the sun from west to east. But the meaning of these terms, west and east, is somewhat different here from their common meaning; for as the earth turns on its axis every twenty four hours, east as we generally understand it, is quite a different direction in space at different times of the day; also, as the earth passes on around the sun, east in its orbit means very different directions at different times of the year. We have, so to speak, to detach our ideas of east and west from our horizon; a rather difficult thing to do, but which the tangent index may perhaps assist in doing.

The earth, in going around the sun, in the course of a year is moving in every direction in the plane of the ecliptic—that is, the earth changes the direction of its motion in space 360 degrees each year, which is nearly one degree per day. But the earth, by turning on its axis in one day, turns our horizon in different successive directions, so that it is puzzling to keep before the mind the true direction. The index helps us in this by the aid of certain quantities, geocentric co-ordinates—namely, the declination of the tangent of the earth's orbit, and its hour angle. Declination has the same meaning as given heretofore. Hour angle, as used here, means the diedral or opening between two great circles of the celestial sphere —namely, the hour circle or meridian, which contains the tangent, and the hour circle of the sun. These quantities vary in value from day to day, but their value depends on the place of the earth in its orbit. As the earth is in nearly the same place in its orbit on a given date year after year, these co-ordinates can be calculated, and their values entered in a table in connection with the corresponding dates. The accompanying table gives these values for dates distributed through the year at sufficiently small intervals; also, on the instrument on the arc of declination and of hour angle are placed dates at intervals of five degrees, from which, by inspection, a suffi-

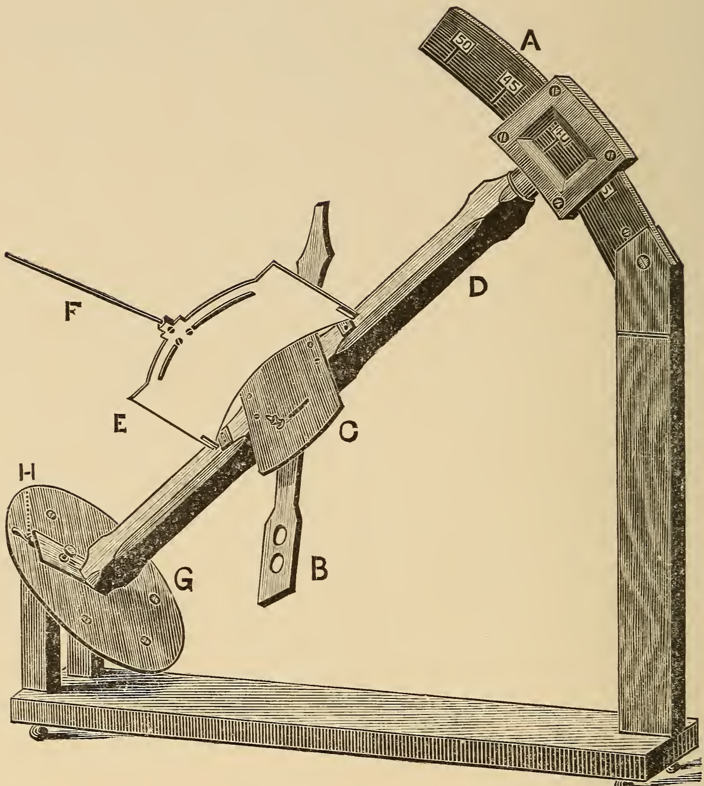
ciently exact estimate of the value for intermediate dates can be made.

To use the instrument, place it on a horizontal table, and in the meridi n, that is, north and south, with the higher end to the north. This can be most conveniently done bv a compass needle. Then raise or lower the north end of the axis so that its elevation shall equal the latitude of he place. The latitude arc is graduated and numbered, so that this is done ea ily. The axis is to be clamped in this position. Next, set the tangent to the proper declination, either by an inspection of the dates on the arc, or by referring to the table; also set the hour angle for the date. Then turn the instrument on the axis till the time index points to the true solar time of day. The tangent index is now a tangent to the earth's orbit, and it points out the direction of the earth's motion in space. Moreover, if the instrument be turned so as to keep up with the time, or if it be turned forward from time to time, it will continue to point the same direction in space the entire twenty-four hours—that is, the eff ct of the earth's diurnal motion is eliminated.

Notice that by the time of day we mean astronomical time—that is, the day begins at noon, and the hours are numbered to twenty-four ; therefore to the A. M. hours we add to our watch or clock time twelve hours. Also, if we w sh to be very exact we take account of the difference between mean and apparent time—that is, we look in the almanac at the date, and if the sun is fast a certain number of minutes, we add that number of minutes to the time given by our timepiece; or if the sun is slow we subtract. If our time is standard (railroad) time, it must first be changed to local time.

The setting of the declination and hour angle need not be changed during the day. Indeed, an inspection of the table will show that the change from one day to the next is mall, only about one-third of a degree at most.

Thus at any hour of the day or night, and on any day of the year, we may exhibit with this instrument with certainty and with all desirable accuracy, the true direction in space of the earth's motion in its orbit.



THE TANGENT INDEX.

A, Latitude arc.
B, Tangent index.
C, Tangent declination arc.
D, Axis of the earth.
E, Solar declination arc.
F, Radius vector.
G, Time circle.
H, Time index and hour angle arc.

CHAPTER IX.

THE TANGENT INDEX TABLE FORMULAS AND DEMONSTRATION.

Table of Declinations and of Hour Angles.

The declinations are found in column D. The sign for any date is found at the head and foot of the column containing the date, + means north declinations, — means south declinations. H is the hour angle. There are two columns of H, and each of these has two columns of corresponding dates:

	H.		D.		H.	
—		+		—		+
March 20	90° 0′	Sept. 23	23° 27′	March 20	90° 0′	Sept. 23
March 25	89 11	Sept. 27	23 22	March 15	90 49	Sept. 18
March 30	88 21	Oct. 3	23 6	March 10	91 39	Sept. 13
April 4	87 35	Oct. 8	22 39	March 5	92 25	Sept. 8
April 9	86 53	Oct. 13	22 1	March 1	93 7	Sept. 3
April 14	86 17	Oct. 17	21 14	Feb. 24	93 43	Aug. 28
April 19	85 46	Oct. 22	20 19	Feb. 19	94 14	Aug. 23
April 24	85 24	Oct. 27	19 13	Feb. 14	94 36	Aug. 18
April 29	85 10	Nov. 1	18 0	Feb. 9	94 50	Aug. 13
May 4	85 4	Nov. 6	16 40	Feb. 4	94 56	Aug. 8
May 9	85 6	Nov. 11	15 13	Jan. 31	94 54	Aug. 3
May 14	85 17	Nov. 16	13 40	Jan. 26	94 43	July 29
May 19	85 36	Nov. 20	12 2	Jan. 21	94 24	July 14
May 24	86 8	Nov. 25	10 20	Jan. 16	93 52	July 19
May 29	86 34	Nov. 30	8 34	Jan. 12	93 26	July 14
June 3	87 12	Dec. 4	6 46	Jan. 7	92 48	July 9
June 8	87 55	Dec. 9	4 55	Jan. 2	92 5	July 4
June 13	88 42	Dec. 14	3 2	Dec. 29	91 18	June 29
June 18	89 30	Dec. 19	1 9	Dec. 24	90 30	June 24
June 21	90 00	Dec. 21	0 0	Dec. 21	90 00	June 21
—		+		—		+

Formulas for computing the numbers in the table:

Sin D = — sin e cos l.

Cos (180 — H) = tan D tan d.

D is the tangent declination.

H is the tangent hour angle.

e is the obliquity of the ecliptic, 23½ degrees, nearly.

l is the Sun's longitude.

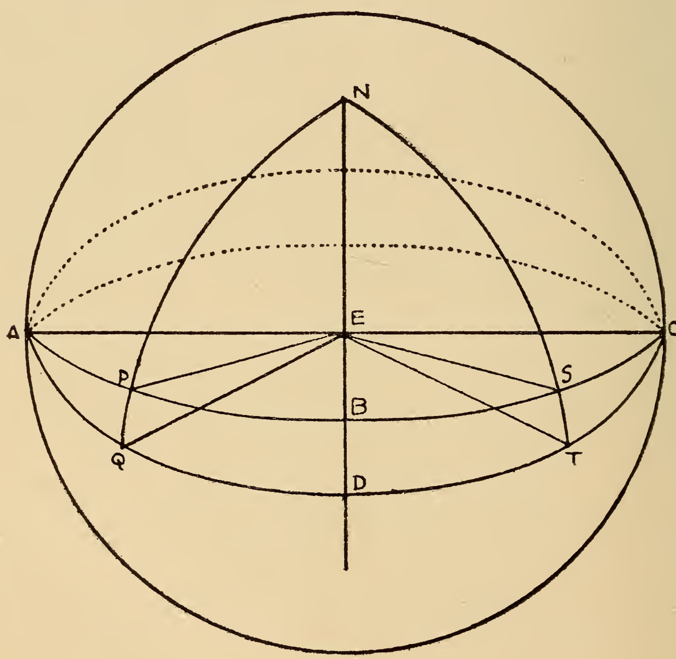
d is the Sun's declination.

e, l and d are taken from the Solar tables, for the date. e may be considered constant.

In computing the table above, the Astronomical Ephemeris, for 1886, was used. The values of l and d were taken at intervals of five days through the quarter, March 21st–June 21st, Greenwich noon of the date. For the other three quarters of the year, the dates were selected with reference to the value of l. Hence, the values of D and H may not be correct to the minute at Greenwich noon; but will represent the values nearly enough for the purpose. As for other years than 1886; while the values in the table are not precise, they will be found exact enough for the purpose of the table for an indefinite number of years.

DEMONSTRATION OF FORMULAS.

The formulas above are demonstrated by the aid of the accompanying diagram:



This represents the celestial sphere. E is the Earth's centre. N E is the axis. N is the north pole. A D C is the equinoctial. A B C is the ecliptic, the Sun's apparent annual path. A is the vernal equinox. The angle B A D is the obliquity of the ecliptic, represented in the formula by e. Let S be the place of the Sun at some time. Then the arc A S is the Sun's longitude, represented by l. Draw S E: this is the radius vector of the orbit. Draw E P in the plane of the ecliptic perpendicular to S E. Then is E P the tangent to the Earth's orbit. The orbit is not represented in the diagram. As the Sun's apparent motion is towards C, the Earth's real motion is in the direction E P. Draw the hour circles N S T and N P Q. S T is the Sun's declination, represented by d, and P Q is the declination of the tangent, represented by D. Also the angle P N S which equals the arc Q T, is the hour angle of the tangent, represented by H. The arc P S is a quadrant, 90 degrees.

In the spherical triangle P A Q, Q a right angle, Sin P Q = sin A P sin P A Q (1). But P A = A S — P S = l — 90° = — (90° — l). Therefore, Sin P A = — sin (90° — l) = — cos l. Substituting in (1) for sin P A this value, and for P Q and P A Q their symbols, we have Sin D = — sin e cos l.

Again, in the quadrantal triangle P N S, P N = 90° — D and N S = 90° — d. It is convenient to solve the triangle polar to P N S, and therefore right angled. The hypothenuse of this triangle is 180° — P N S or 180° — H. The adjacent angles are 180° — P N and 180° — N S. That is 180° — (90° — D) = 90° + D for the first; and 180° — (90° — d) = 90° + d for the second. Then cos (180° — H) = cot (90° + D) cot (90° + d) = (— tan D) (— tan d) = tan D tan d.

We may also find H = Q T by first computing A Q in the triangle P A Q. Then Q T = A T — A Q. A T is the Sun's right ascension. Calling this a, and calling A Q, the right ascension of the tangent, A, we have H = a — A. The formula for A is obtained thus: $\text{Tan } A\,Q = \frac{\tan A\,P}{\cos P\,A\,Q} = \frac{\tan (A\,S - P\,S)}{\cos P\,A\,Q}$. That is $\tan A = \frac{\tan (l - 90^\circ)}{\cos e} = \frac{-\tan (90^\circ - l)}{\cos e} = -\frac{\cot l}{\cos e}$.

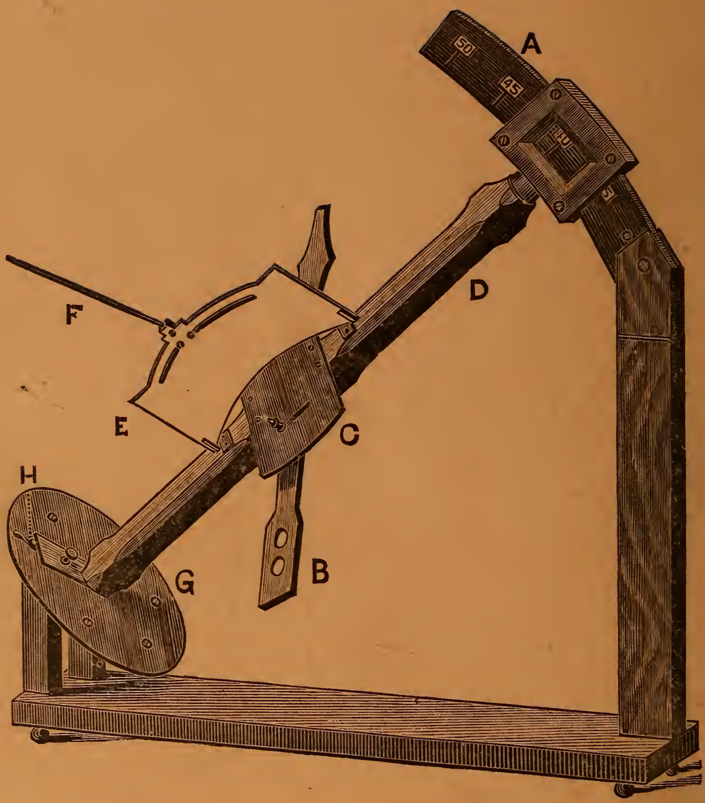
It can be seen, either from the formula, or from the diagram, that D varies in value between the limits — e and + e; the former limit being the value of D when the Sun is at the vernal equinox; the

latter being the value of D when the Sun is at the autumnal equinox. Its value is 0° when the Sun is at either solstice.

Also, it is seen that H has the value 90° when D is 0°, which is the case as stated above, when the Sun is at either solstice; or when d is 0°, which is the case when the Sun is at either equinox. About midway between the times for these values, H has a minimum value of 85° 4′, viz., about May 4th and November 6th; and a maximum value of 94° 56′, viz., about February 4th and August 8th. These facts can also be seen from an inspection of the table.

The analysis above is based upon the condition that the tangent to the Earth's orbit, and the radius vector to the point of contact are perpendicular to each other; that is, that the orbit is a circle with the Sun at the center. But the tangent is perpendicular to the radius vector only at perihelion and aphelion; that is, about January 1st and July 1st. The formulas therefore are strictly true only for these times, and only approximately true for other times. But on account of the small eccentricity of the orbit, the angle between the tangent and radius vector differs from 90° less than a degree at the most. Thus, on April 1st, when the Earth is at its mean distance from the Sun, the forward end of the tangent makes with the radius vector an angle of 90° 58′ nearly. And, again, about October 2nd the angle is about 89° 2′. These are the extreme values. If the exact value of this angle were used for the several dates, in computing the values of D and H, these would be found to differ from those given in the table only a few minutes in any case. The numbers in the table may therefore be considered sufficiently exact for use in connection with the Tangent Index.

The discussion of the astronomical relations of the Earth, according to the scheme proposed at the beginning, is now completed. The presentation of the general plan of the Solar System was not in the scheme; neither the consideration of the perturbations, the inequalities in motion caused by the mutual gravity of the several members of the system. These effects are suitably discussed in many excellent works on Astronomy, to which the student is referred for information on these subjects. The end proposed in this discussion is accomplished if the form and principal motions of the Earth are more clearly seen.



THE TANGENT INDEX.

A Device of the Author.